Exercise 14

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$y(x) = 2x^2 + 10x + 12$$

Solution

Begin by factoring the coefficient of x^2.

$$y(x) = 2\left(x^2 + 5x + 6\right)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 5$, which means $B = \frac{5}{2}$ and $B^2 = \frac{25}{4}$. Add and subtract $\frac{25}{4}$ on the right side within the parentheses and use the identity so that x appears in only one place.

$$y(x) = 2\left[\left(x^2 + 5x + \frac{25}{4}\right) + 6 - \frac{25}{4}\right]$$

$$= 2\left[\left(x + \frac{5}{2}\right)^2 - \frac{1}{4}\right]$$

$$= 2\left(x + \frac{5}{2}\right)^2 - \frac{1}{2}$$

Therefore, the vertex of the parabola is $\left(-\frac{5}{2}, -\frac{1}{2}\right)$. The axis of symmetry is $x = -\frac{5}{2}$, and the minimum (because the coefficient of x^2 is positive) is $y = -\frac{1}{2}$.